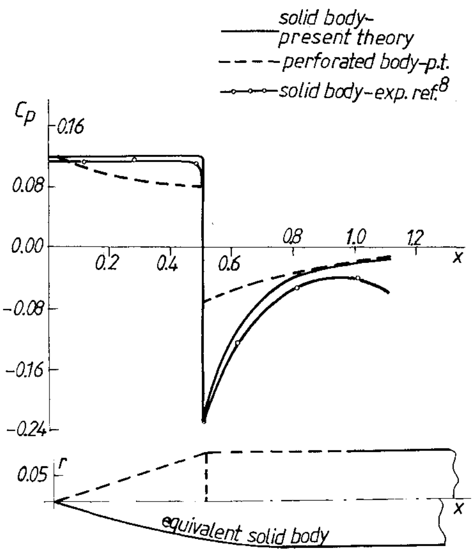
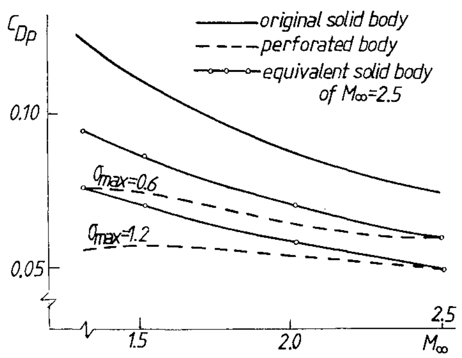



a) Pressure coefficient distribution.




b) Drag variation vs Mach numbers.

**Fig. 3 Cone-cylinder ogive with $\theta^* = 10$ deg at $M_\infty = 1.32$.**

This study can be improved by assuming a more complex physical model and/or by using a more accurate theory. On the other hand, experimental data are expected to be in good agreement with the results presented herein.

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# Transient Temperature Distributions in a Radiantly Heated Hollow Cylinder

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## Introduction

A HOLLOW cylinder is a common structural element in a spacecraft, e.g., in booms, struts, and antennas. Therefore, the thermal behavior of such an element in the space environment is of some interest, especially since the circumferential temperature distribution affects its deflection and state of stress. Parts of the problem have been addressed previously. Olmstead[1] treated the transient response 20 years ago without regard for internal radiation. Steady-state analyses were made by Charnes and Raynor[2] and Nichols[3] with allowance for wall conduction and external radiation, but for no internal heat exchange. Sparrow and Krowech[4] treated internal convection into an opaque fluid. Hrycak and Helgans[5] considered internal radiation correctly for a black interior and neglected radial thermal resistance. Edwards[6] allowed for radial resistance and a nonblack wall.

This Note treats the transient temperature distribution around a spinning hollow thin-walled cylinder with arbitrary initial temperature and allowance for internal and external thermal radiation transfer. Sample calculations show the effect of time from startup and spin rate on the temperature profile.

## Formulation

A cylinder of mean radius $R$ and thin-wall thickness $b$ is shown in Fig. 1. For the thin wall, the wall temperature may be assumed to be uniform over thickness. The heat conduction equation with radiative boundary conditions is then written as a fin equation:

$$\rho c b \frac{\partial T}{\partial t} = \frac{kb}{R^2} \frac{\partial^2 T}{\partial \theta^2} - (q_e + q_i) + q_a \tag{1}$$

Here $q_e$ is the externally emitted heat flux, $\epsilon_e \sigma T^4$ in deep space, $q_i$ is the net heat flux into the interior, and $q_a$ the externally absorbed flux. The net internal flux is the difference between the radiosity $q^+$ and irradiation $q^-$

$$q_i = q^+ - q^- \tag{2}$$

where

$$q^+ = \epsilon_i \sigma T^4 + (1 - \epsilon_i) q^- \tag{3}$$

$$q^-(\theta) = \int_{-\pi}^{+\pi} q^+(\theta') K(\theta, \theta') \mathrm{d}\theta' \tag{4}$$

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The kernel is

$$K(\theta,\theta') = \tfrac{1}{4}\sin(|\theta-\theta'|/2) \qquad (5)$$

For direct solar irradiation, the externally absorbed flux is

$$q_a = \alpha_s G_s \cos\beta \cos(\theta-\omega t) U(\theta-\omega t) \qquad (6)$$

where $\alpha_s$ is the solar absorptivity, $G_s$ the solar constant referred to the appropriate distance from the sun, $\beta$ the angle off of normal to the cylindrical axis, and $U$ the unit step function. The angle $\theta$ is measured from a fixed arbitrary line around the cylinder wall, and time $t=0$ is taken when that line passes through the solar stagnation point. Sunlight or shade is accounted for by the unit step function

$$U(\theta-\omega t) = 1 \qquad -\pi/2 < (\theta-\omega t) < \pi/2$$

$$= 0 \qquad \pi/2 < |\theta-\omega t| < \pi \qquad (7)$$

Linearization is usually acceptable because the absolute temperature variations are small.

$$\sigma T^4 = \sigma T_0^4 + 4\sigma T_0^3 (T-T_0) \qquad (8)$$

It is most convenient to transform to the angular coordinate measured from solar stagnation, thus to a moving coordinate with respect to a point on the cylinder,

$$\tilde{\theta} = \theta - \omega t \qquad (9)$$

and to use dimensionless variables and parameters. These are

$$T^* = (T-T_0)/T_0 \qquad (10)$$

$$q^* = (q^- - \sigma T_0^4)/\sigma T_0^4 \qquad (11)$$

$$t^* = t/\tau \qquad (12)$$

where

$$\tau = (b\rho c)/(\epsilon_e \sigma T_0^3) \qquad (13)$$

$$\omega^* = \omega\tau \qquad (14)$$

$$k^* = (kb)/(R^2 \epsilon_e \sigma T_0^3) \qquad (15)$$

$$G^* = (q_a - q_e)/\epsilon_e \sigma T_0^4 \qquad (16)$$

The governing equations become

$$\frac{\partial T^*}{\partial t^*} - \omega^* \frac{\partial T^*}{\partial \tilde{\theta}} = k^* \frac{\partial^2 T^*}{\partial \tilde{\theta}^2} + G^* - 4\left(\frac{\epsilon_i}{\epsilon_e} + 1\right) T^* + \frac{\epsilon_i}{\epsilon_e} q^* \qquad (17)$$

$$q^*(\tilde{\theta},t^*) = \int_{-\pi}^{+\pi} [4\epsilon_i T^*(\tilde{\theta}',t^*) + (1-\epsilon_i) q^*(\tilde{\theta}',t^*)] K(\tilde{\theta},\tilde{\theta}') d\tilde{\theta}' \qquad (18)$$

It is convenient to express $G^*$ in a Fourier series and choose $T_0$ so that the first term $a_0$ is zero.

$$G^* = \sum_{n=1}^{\infty} a_n \cos n\tilde{\theta} \qquad (19)$$

$$a_n = \frac{2}{\pi} \int_0^{\pi/2} \frac{\alpha_s(\theta) G_s \cos\beta \cos\tilde{\theta} \cos n\tilde{\theta}}{\epsilon_e \sigma T_0^4} d\tilde{\theta} - \frac{2}{\pi} \int_0^{\pi} \cos n\tilde{\theta} d\tilde{\theta} \qquad (20)$$

$$\sigma T_0^4 = \frac{1}{\pi} \int_0^{\pi/2} (\alpha_s/\epsilon_e) G_s \cos\beta \cos\tilde{\theta} d\tilde{\theta} \qquad (21)$$

Thus, the temperature $T_0$ is the usual equilibrium temperature for a perfectly conducting cylinder affected strongly by the $\alpha_s/\epsilon_e$ ratio.

Superposition applies to Eqs. (17) and (18). One can write the general solution as the sum of steady and transient components as

$$T^*(\tilde{\theta},t^*) = T_1^*(\tilde{\theta}) + T_2^*(\tilde{\theta},t^*) \qquad (22)$$

$$q^*(\tilde{\theta},t^*) = q_1^*(\tilde{\theta}) + q_2^*(\tilde{\theta},t^*) \qquad (23)$$

Equations (17) and (18) admit steady solutions in the form

$$T_1^* = \sum_{n=1}^{\infty} (b_n \cos n\tilde{\theta} + b_n' \sin n\tilde{\theta}) \qquad (24)$$

$$q_1^* = \sum_{n=1}^{\infty} (c_n \cos n\tilde{\theta} + c_n' \sin n\tilde{\theta}) \qquad (25)$$

where

$$\frac{c_n}{b_n} = c_n' = \frac{4\epsilon_i S_n}{1-(1-\epsilon_i)S_n} \qquad (26)$$

$$4S_n = \frac{1}{\cos n\theta} \int_{-\pi}^{+\pi} K(\theta,\theta') \cos n\theta' d\theta'$$

$$= \frac{1}{\sin n\theta} \int_{-\pi}^{+\pi} K(\theta,\theta') \sin n\theta' d\theta' \qquad (27)$$

$$S_n = \frac{-1}{4n^2-1} \qquad (28)$$

$$b_n = \frac{a_n d_n}{d_n^2 + n^2 \omega^{*2}} \qquad (29)$$

$$b_n' = -\frac{n\omega^* b_n}{d_n} \qquad (30)$$

$$d_n = k^* n^2 + 4[1 + (\epsilon_i/\epsilon_e)(1-S_n)][1-(1-\epsilon_i)S_n]^{-1} \qquad (31)$$

The transient solutions are

$$T_2^*(\tilde{\theta},t^*) = \sum_{n=1}^{\infty} (B_n \cos n\tilde{\theta} + B_n' \sin n\tilde{\theta}) e^{\lambda_n t^*} \qquad (32)$$

$$q_2^*(\tilde{\theta},t^*) = \sum_{n=1}^{\infty} (C_n \cos n\tilde{\theta} + C_n' \sin n\tilde{\theta}) e^{\lambda_n t^*} \qquad (33)$$

where

$$\lambda_n = -d_n \pm i n\omega^* \qquad (34)$$

The coefficients depend upon the initial conditions.

## Solution for Startup of Rotation

One case of interest is a cylinder deployed into space, allowed to heat to an equilibrium condition, and then rotated. The initial temperature distribution follows from setting $\omega^* = 0$.

$$T^*(\theta,0) = \sum_{n=1}^{\infty} (a_n/d_n) \cos n\tilde{\theta} \qquad (35)$$

By equating Eq. (32) to Eq. (35) at $t^* = 0$, coefficients $B_n$ and $B_n'$ are found. The solution is

$$T^* = \sum_{n=1}^{\infty} \{a_n \left[\frac{d_n \cos n\tilde{\theta} - n\omega^* \sin n\tilde{\theta}}{d_n^2 + n^2\omega^{*2}}\right]$$

$$- a_n \left[\frac{d_n \cos n(\tilde{\theta}+\omega^* t^*) - n\omega^* \sin n(\tilde{\theta}+\omega^* t^*)}{d_n^2 + n^2\omega^{*2}}\right] e^{-d_n t^*}$$

$$+ (a_n/d_n)\cos n(\tilde{\theta}+\omega^* t^*) e^{-d_n t^*} \} \tag{36}$$

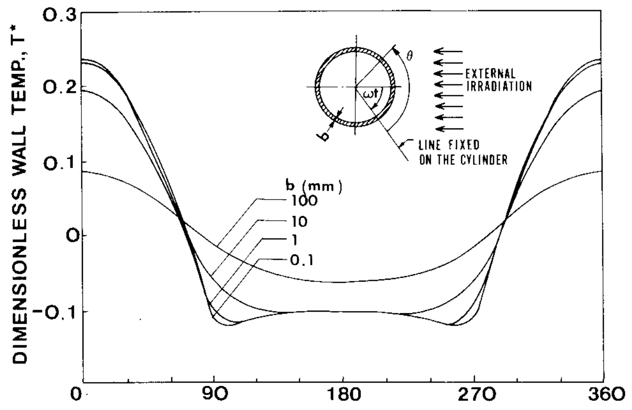


**Fig. 1 The effect of wall thickness on initial temperature distribution.**

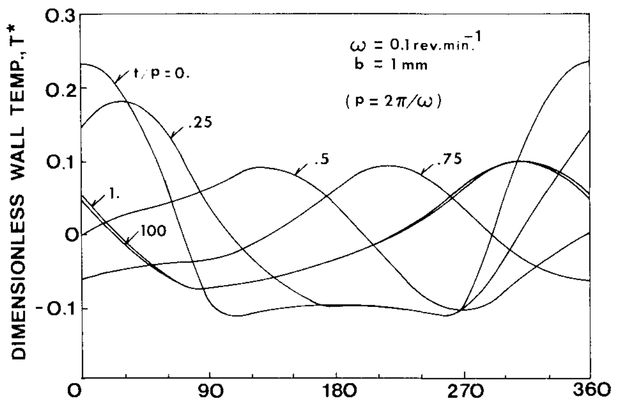


**Fig. 2 Temperature response after the initiation of spinning.**

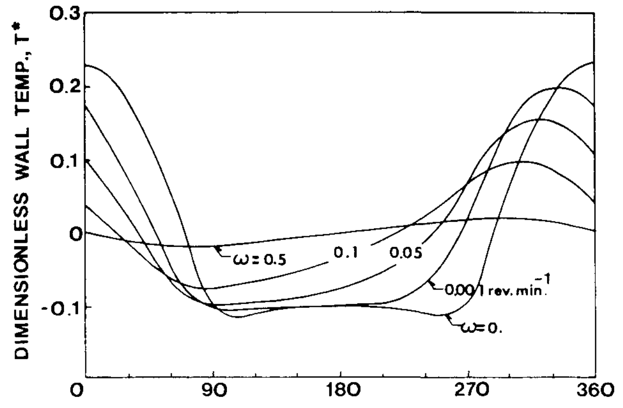


**Fig. 3 The effect of angular velocity on quasisteady temperature distribution.**

For example, a black aluminum cylinder with a 1-m mean radius is considered for the following conditions:

$$\alpha_s = \epsilon_e = \epsilon_i = 1.0, \qquad k = 177 \text{ W/m-K}$$

$$\rho = 2770 \text{ kg/m}^3, \qquad c_p = 875 \text{ J/kg-K}$$

Figure 1 shows the initial temperature profiles for values of cylinder wall thickness $b$ ranging from 0.1 to 100 mm. Figure 2 shows the transient response for the radiation-dominant case, $b = 1$ mm. Shortly after spinning initiates, the peak temperature drops rather rapidly. The hottest point is shifted in the direction of rotation toward the solar terminator. A value of $\omega t/2\pi = 1$, i.e., the time to turn just one revolution is enough to establish very nearly the new temperature profile.

Figure 3 shows the effect of angular velocity $\omega$ on the aysmptotic temperature profile as $t \to \infty$. At high velocity just a simple sinusoidal temperature profile becomes established with the amplitude reduced by increased rotation.

These calculations are obtained from an eight-term approximation of Eq. (36). However, only a two-term approximation is needed for engineering accuracy.

## Solution for Axial Stripes

Axial stripes can be readily accommodated merely by recognizing that $\alpha_s$ can vary spatially with $\theta$. For example, with two different diffuse coatings with solar absorptivity, $\alpha_{s1}$ and $\alpha_{s2}$, the coefficients $a_n$ can be written as

$$a_n = \frac{2}{\pi} \frac{G_s \cos\beta}{\epsilon_e \sigma T_0^4} \left\{ \int_0^{\Delta\theta/2} \alpha_{s1} \cos\theta \cos n\theta \, d\theta \right.$$

$$\left. + \int_{\Delta\theta/2}^{\Delta\theta/2+\Delta\theta} \alpha_{s2} \cos\theta \cos n\theta \, d\theta + \ldots \right\} \tag{37}$$

where $\Delta\theta$ is the angular width of the stripe.

## Conclusion

In conclusion, Eq. (36) together with Eqs. (20), (28), and (31) permit the calculation of temperature as a function of time and position around a stationary or rotating hollow cylinder.

It was shown in a realistic example that quasisteady state was achieved within only one revolution after an impulsive startup of rotation. The solution for steady rotation, Eq. (24), requires only two terms for engineering accuracy, and Fig. 3 shows that rotation somewhat in excess of a few revolutions per minute will effectively eliminate circumferential temperature gradients.

## Acknowledgment

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